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Sensitivity and specificity of coherence and phase synchronization analysis

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Abstract

In this Letter, we show that coherence and phase synchronization analysis are sensitive but not specific in detecting the correct class of underlying dynamics. We propose procedures to increase specificity and demonstrate the power of the approach by application to paradigmatic dynamic model systems.

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1. Introduction

Several approaches and strategies exist for analyzing the great variety of time series generated by multivariate dynamic processes

$$\dot{\mathbf{x}}(t) = \mathbf{f}\big(\mathbf{x}(t), \mathbf{p}(t), \boldsymbol{\eta}(t)\big). \tag{1}$$

The multivariate process is denoted by $\mathbf{x}(t)$, $\mathbf{p}(t)$ is a set of parameters, and $\boldsymbol{\eta}(t)$ a multidimensional noise process. The type of dynamics and the interaction structure between the processes is modeled by the function $\mathbf{f}$. In the following, we concentrate on two main classes of processes: non-linear deterministic and linear stochastic processes [1,2]. For both classes different tools have been developed in order to analyze time series generated by these processes. These analysis techniques have been widely applied to empirical time series [3–18].

In the non-linear deterministic case, systems of coupled self-sustained oscillators are examined. Coupled self-sustained oscillators are able to synchronize [19]. Apart from oscillators characterized by a limit cycle behavior, chaotic oscillators attracted particular interest over the last years. Based on the different synchronization regimes gained by coupled chaotic oscillators, the further analyses concentrate on these oscillators. Increasing the coupling strength between chaotic non-identical oscillators can lead to a transition from unsynchronized to phase synchronized oscillators [20]. In addition, phase synchronization is characterized by almost uncorrelated amplitudes since their correlation increases slow compared to the phase correlation. Phase synchronization precedes a lag and almost complete synchronization accompanied by highly correlated amplitudes [21]. Since in applications data are influenced by noise, the underlying theory of non-linear deterministic dynamics has been extended to stochastic systems [8].

In the linear stochastic case, transfer function systems are investigated [1]. Transfer functions represent filters relating processes. While processes and filters initially were assumed

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to be linear, linearity of processes is not necessary, and the corresponding analysis techniques have been extended to certain types of non-linear transfer functions and feedback systems [22,23].

It is important to note that systems from these two classes show different dynamic behavior. Transfer function systems are characterized by a clearly defined input and output. The output signal ceases to exist in absence of an input. For synchronizing systems, however, there is no input and output since both oscillators are self-sustained and continue oscillating independently in absence of any coupling. Thus, coupled self-sustained oscillators are able to synchronize which is impossible for transfer function systems.

For either systems, analysis techniques have been developed which have been shown to be *sensitive* in detecting the dynamic properties if the assumed class of processes is present. For instance, coherence analysis is able to detect interactions in transfer function systems. This procedure is referred to as the so-called *direct problem*.

However, in application to empirical data, one faces an *inverse problem*, since the underlying dynamics generating the data is not known in advance. For instance, it is not possible to unequivocally conclude a transfer function system from a significant coherence spectrum. Therefore, it is desirable to apply analysis techniques that are *specific* in detecting these different dynamic properties only if the assumed class of processes is present.

The established analysis techniques coherence and phase synchronization are widely used in applications, for instance in neuroscience to electroencephalographic recordings. When utilized to learn neurophysiological or pathophysiological mechanisms, knowledge of the specificity of the time series analysis techniques applied is essential. In this Letter, we show that coherence and phase synchronization are sensitive but not specific in detecting the correct class of underlying dynamics generating the time series investigated.

A coupled stochastic Roessler system is used as an example for the class of coupled self-sustained oscillators. As a representative for transfer function systems, the x-component of a Roessler oscillator is time-delayed and propagated through a first order low-pass filter. These two classes of dynamic systems are introduced in Section 2. In Sections 3 and 4, we demonstrate sensitivity and lack of specificity of phase synchronization and coherence analysis for the systems under investigation. To overcome the limitation of low specificity of coherence and phase synchronization analysis, we propose a combination of both methods and a detailed analysis of the phase signals in Section 5. For the paradigmatic model systems investigated, we show that a specific inference of the underlying dynamics is possible by the proposed methodology.

2. Two classes of dynamics

In the following subsections, examples of the two classes, namely coupled self-sustained non-linear oscillators and linear transfer function systems, are introduced.

2.1. Coupled self-sustained oscillators

As a representative of coupled self-sustained and therefore necessarily non-linear oscillators, the stochastic extension of a coupled Roessler system with frequencies $\omega_{1,2}$ [24]

$$\begin{aligned}
\dot{x}_{1,2} &= -\omega_{1,2}y_{1,2} - z_{1,2} + \varepsilon_{1,2}(x_{2,1} - x_{1,2}) + \sigma_{1,2}\eta_{1,2},\\
\dot{y}_{1,2} &= \omega_{1,2}x_{1,2} + ay_{1,2},\\
\dot{z}_{1,2} &= b + (x_{1,2} - c)z_{1,2}
\end{aligned} \quad (2)$$

is investigated. Coupling strength and direction between the two oscillators are determined by the parameters $\varepsilon_{1,2}$. Dynamic noise influence is modeled by Gaussian distributed random variables $\eta_{1,2} \sim \mathcal{N}(0,1)$ leading to the variance $\sigma_{1,2}^2$ of the noise term $\sigma_{1,2}\eta_{1,2}$. Using $a = 0.15$, $b = 0.2$, $c = 10$, and $\omega_{1,2} = 1 \pm 0.015$ Hz leads to chaotic oscillations for the deterministic system [20] (cf. Fig. 1(a)).

2.2. Transfer function systems

As a representative of a transfer function system, we investigate a first order low-pass filtered and time-delayed propagated signal

$$u(t) = a_1u(t-1) + a_2x(t-\tau) + \sigma\tilde{\eta}(t), \quad (3)$$

where $x(t)$ is the x-component of a stochastic Roessler system with $a = 0.15$, $b = 0.2$, $c = 10$, and $\omega = 1$ (Fig. 1(b)). The low-pass filter is realized by the autoregression of order one with parameter $0 \leqslant a_1 < 1$, since the spectrum of process (3) is given by

$$S_u(\omega) = \frac{S_{a_2x+\sigma\tilde{\eta}}(\omega)}{|1 - a_1e^{-i\omega}|^2} \overset{\omega \ll 1}{\approx} \frac{S_{a_2x+\sigma\tilde{\eta}}(\omega)}{(1+a_1)^2 + a_1\omega^2}, \quad (4)$$

where $S_{(\cdot)}$ denotes the spectrum of process $(\cdot)$. The spectrum of the output signal $u(t)$ shows that small frequencies are weighted with a higher amplitude, which characterizes a low-pass filter. The magnitude and lag of the time-delayed influence are quantified by the parameters a_2 and τ, respectively. $\sigma\tilde{\eta}(t)$ represents additional stochastic influences modeled by uncorrelated Gaussian noise $\tilde{\eta} \sim \mathcal{N}(0,1)$.

The major difference between system (2) and system (3) can be illustrated by choosing $\varepsilon_{1,2} = 0$ and $a_2 = \sigma = 0$, respectively. As the Roessler oscillators are self-sustained, both continue oscillating independently, while in the case of the transfer function system, the output signal $u(t)$ decreases exponentially for arbitrary initial conditions.

In the following investigations, the standard deviation of the noise is chosen to be $\sigma_{1,2} = 1.5$ for the coupled Roessler system and $\sigma = 1.5$ for the transfer function system, respectively.

3. Phase synchronization

To detect phase synchronization of coupled non-identical chaotic oscillators with $\omega_{1,2} = 1 \pm \Delta\omega$, phase and amplitude of the real-valued signal have to be considered. Several approaches for calculating phase and amplitude of a signal were proposed [20,25,26]. In what follows the definition based on

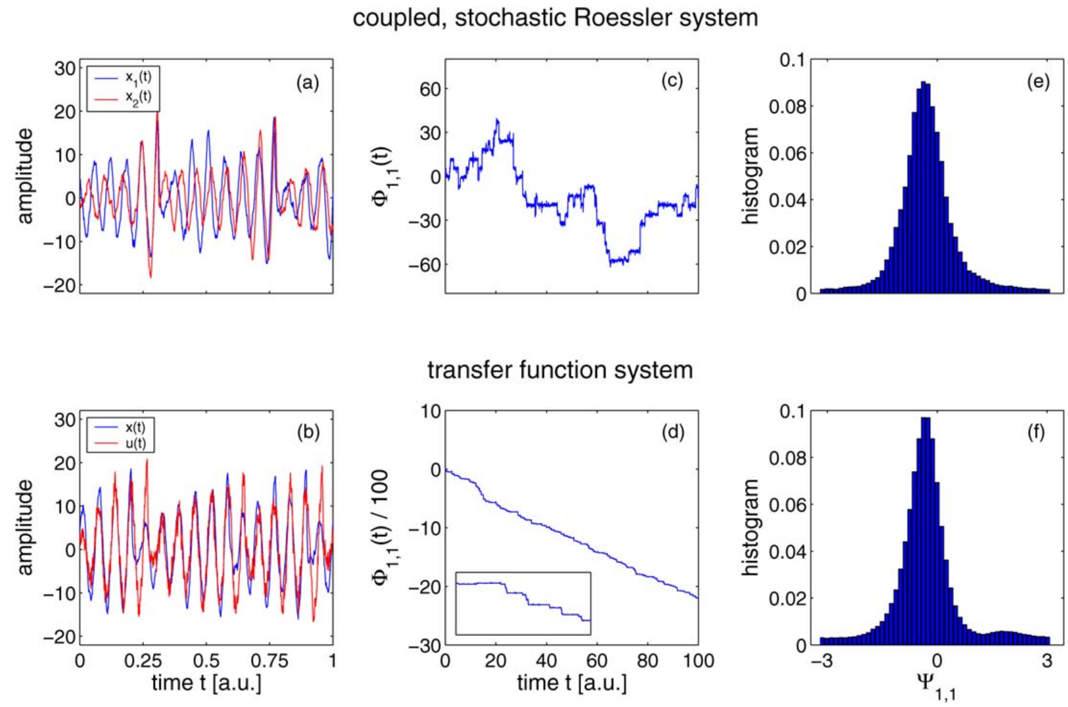

Fig. 1. Time series of a coupled stochastic Roessler system (a) and of the considered transfer function model ($\tau = 64$) (b). Phase difference $\Phi_{1,1}$ for the coupled stochastic Roessler system (c) and for the transfer function model (d). The small subplot in (d) shows a section of the phase difference of two time units duration more closely. Considering the temporal evolution of the phase differences $\Phi_{1,1}$, rapid phase changes in one direction are preferred for the transfer function model. For the coupled stochastic Roessler system, phase jumps in both directions occur with almost the same frequency in this example. Corresponding normalized histograms of $\Psi_{1,1}$ in (e) and (f). Even if both systems possess a different dynamic behavior, a preferred value of $\Psi_{1,1}$ is taken for both. A differentiation between both classes of dynamics is impossible by just applying phase histogram analysis.

Gabor's analytic signal representation [27]

$$\psi(t) = x(t) + i\hat{x}(t), \tag{5}$$

expressed via its polar representation

$$\psi(t) = A(t)e^{i\Phi(t)}, \tag{6}$$

where $A(t)$ denotes amplitude and $\Phi(t)$ phase of the analytic signal, is used. The Hilbert transform [28]

$$\hat{x}(s) = \frac{1}{\pi}\,\mathrm{P.V.}\int x(t)\frac{1}{s-t}\,\mathrm{d}t \tag{7}$$

yields the imaginary counterpart $\hat{x}(t)$ of the real-valued observed signal $x(t)$; P.V. refers to Cauchy's principal value. This procedure is reasonable for oscillatory signals with an almost clearly defined single frequency [29]. Broad-band signals or signals with more frequency components are band-pass filtered in order to apply the analytic signal approach.

If the phase locking condition [20]

$$\left|n\Phi^{(1)} - m\Phi^{(2)}\right| = |\Phi_{n,m}| < \text{const}, \tag{8}$$

is fulfilled for two non-identical oscillators, where $\Phi^{(i)}$ denotes the phase of time series i and $\Phi_{n,m}$ the phase difference for given integers n and m, these oscillators are referred to as n:m phase synchronized. In the presence of additional stochastic influence, phase jumps of $\pm 2\pi, \pm 4\pi, \ldots$ occur. Therefore, the distribution of

$$\Psi_{n,m} = \Phi_{n,m} \bmod 2\pi \tag{9}$$

is investigated. For two phase synchronized processes, a sharp peak in the histogram of the phase differences is observed [8]. A synchronization index, quantifying the sharpness of these peaks, is given by [30,31]

$$R_{n,m}^2 = \left\langle \cos\Psi_{n,m}(t)\right\rangle^2 + \left\langle \sin\Psi_{n,m}(t)\right\rangle^2. \tag{10}$$

This synchronization index is normalized with $R_{n,m} = 1$ indicating a constant phase difference and $R_{n,m} = 0$ for uniformly distributed phase differences.

3.1. Sensitivity

Since phase synchronization analysis has been developed for detection of weak coupling between self-sustained oscillators, to investigate sensitivity, in Fig. 1(c) the phase difference $\Phi_{1,1}$ calculated for the $x_{1,2}$-components of the Roessler system (2) with a frequency mismatch $2\Delta\omega = 0.03$ is investigated. The bidirectional coupling is chosen to be $\varepsilon_{1,2} = 0.1$. Several phase jumps of 2π are observed while the phase difference in between is almost constant. This leads to a sharp peak in the distribution of $\Psi_{1,1}$, illustrated in Fig. 1(e). The synchronization index $R := R_{1,1} = 0.76$ substantiates the presence of phase synchronization in this case.

3.2. Specificity

To investigate specificity, phase synchronization analysis is applied to the transfer function system, Eq. (3), with $a_1 = 0.3$

and $a_2 = 0.7$ using the x-component of a stochastic Roessler oscillator as input process. In Fig. 1(d) the phase difference $\Phi_{1,1}$ between the input process x and the output process u (cf. Eq. (3)) and in Fig. 1(f) the corresponding distribution of $\Psi_{1,1}$ is shown. Due to the sharp peak, a phase synchronization between the two processes is also strongly indicated. But the input signal and the filtered and propagated output signal do not fulfill the necessary conditions for phase synchronization as they are not self-sustained. Thus, quantifying the histogram of the phase difference is not a specific analysis technique and it is therefore not possible to conclude the type of underlying dynamics by just investigating the distribution of the phase difference $\Psi_{1,1}$.

4. Coherence analysis

A classic analysis technique for detection of linear relationships in transfer function systems is coherence analysis. Briefly, coherence is the Fourier domain counterpart of the cross-correlation function. Coherence between two processes x_i and x_j is defined as

$$\mathrm{Coh}_{ij}(\omega) = \frac{|CS_{ij}(\omega)|}{\sqrt{S_{ii}(\omega)S_{jj}(\omega)}}, \tag{11}$$

while

$$S_{ii}(\omega) = \mathcal{FT}\{ACF_{ii}(\tau)\} \tag{12}$$

and

$$CS_{ij}(\omega) = \mathcal{FT}\{CCF_{ij}(\tau)\}, \quad i \neq j. \tag{13}$$

$\mathcal{FT}\{\cdot\}$ denotes the Fourier transformation, ACF the auto-covariance, and CCF the cross-covariance function. $S_{ii}(\omega)$ denote the auto- and $CS_{ij}(\omega)$ the cross-spectra. Coherence is normalized in the unit interval [0, 1], while a coherence value of 1 is observed for a perfect linear relationship between the processes. For a α-significance level, absence of a linear relationship between the two processes can be deduced from a coherence value below a critical value

$$s = \sqrt{1 - \alpha^{\frac{2}{\nu - 2}}}. \tag{14}$$

The equivalent number of degrees of freedom ν depends on the estimation procedure [23,32–36].

4.1. Sensitivity

Since coherence analysis has been developed to detect interactions in linear transfer function systems, to investigate sensitivity, coherence analysis is shown for our transfer function model, Eq. (3), in Fig. 2(a). The critical value for a 1%-significance level is marked by the vertical line. Especially at the oscillation frequency (dotted line), a highly significant coherence is observed. It reflects the transfer function system simulated and indicates sensitivity of the analysis technique. The almost vanishing coherence for higher frequencies is due to the filter properties and the additional stochastic noise influence.

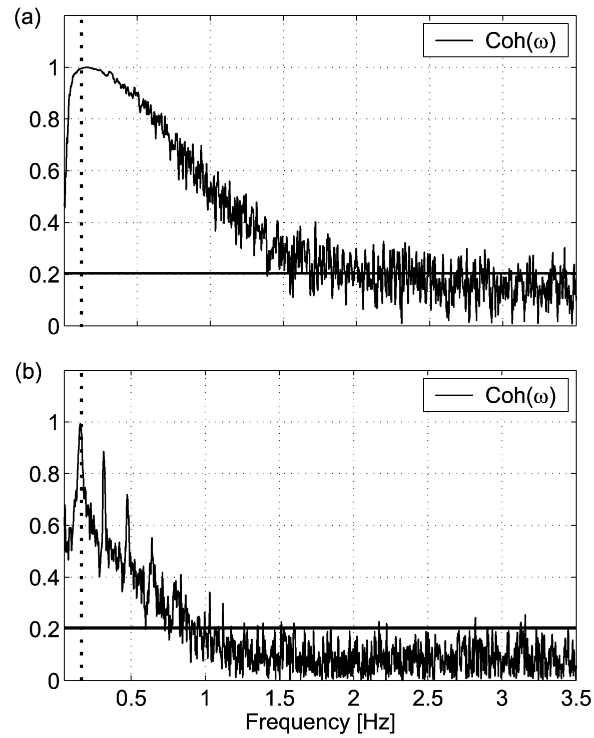

Fig. 2. Coherence between the x-component of a Roessler oscillator and its propagated signal u (a) and coherence between the x_1- and x_2-components of a bidirectionally coupled stochastic Roessler system (b). The dotted vertical line indicates the oscillation frequency of the input oscillator in (a) and the common frequency of the synchronized oscillators in (b). Since in both figures coherence exceeds the critical values for a 1%-significance level, given by the solid, horizontal line, a linear relationship is strongly indicated. A differentiation between both classes of dynamic systems is impossible by just applying coherence analysis.

4.2. Specificity

To investigate specificity, coherence analysis is applied to the coupled stochastic Roessler system Eq. (2). The critical value for a 1%-significance level is marked by the vertical line. A highly significant coherence is detected between the x-components, cf. Fig. 2(b). Thus, coherence analysis is not specific in detecting linear transfer function systems.

5. Approach to increase specificity

In this section, we propose for both, coherence and phase synchronization analysis, an approach to increase specificity in detecting the type of underlying dynamics. The first step is to investigate the direction of phase slips. Second, coherence analysis is applied to the fluctuations of the phase signals.

5.1. Direction of phase jumps

In Fig. 1(c), phase jumps occur in both directions with almost the same frequency for the example of a bidirectionally coupled stochastic Roessler system. In contrast in Fig. 1(d), phase changes in one direction are preferred for the transfer function model under investigation. Due to the rapidity of these phase changes, caused by trajectories of the analytic signal

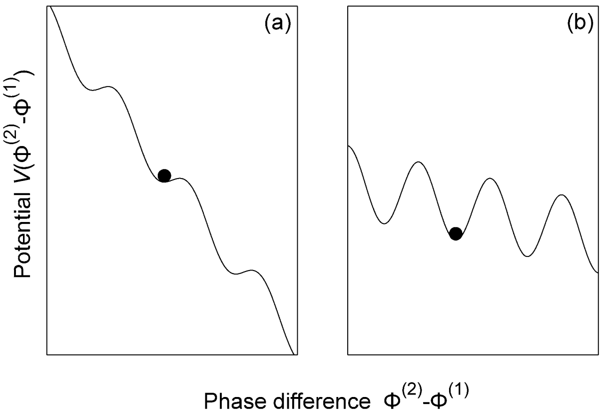

Fig. 3. Washboard potential of the phase difference for two different parameter choices. In (a) a low coupling strength and a high frequency mismatch between the Roessler systems is chosen. In (b) the frequency mismatch is chosen to be low compared to the high coupling. The stochastic influence may lead to jumps of the phase difference in both directions with almost the same probability in (b). In contrast, phase jumps in the positive horizontal direction are more likely in (a).

close to the origin of the phase space, they are not distinguishable from phase jumps between synchronized self-sustained oscillators and are therefore also referred to as phase jumps in the following.

In order to explain the difference in the directionality of phase jumps for the two classes (cf. Fig. 1), an approximation of the potential for the phase difference [19]

$$V(\Phi_{1,1}) = 2\Delta\omega\Phi_{1,1} + \varepsilon\cos(\Phi_{1,1}) \tag{15}$$

is considered for coupled oscillatory systems. The corresponding potential is outlined in Fig. 3(a) and (b). In Fig. 3(a) the coupling strength is chosen to be low compared to the frequency mismatch. In Fig. 3(b) the potential for a highly synchronized state is shown. For the stochastic Roessler system under investigation, the noise influence leads to jumps of the phase difference. According to the different steepness of the potentials for the phase difference, phase jumps in one direction are preferred for high frequency mismatches and low coupling strengths (cf. Fig. 3(a)) or phase jumps in both directions are observed with almost the same probability for highly synchronized states (cf. Fig. 3(b)). However, for a high frequency mismatch and a low coupling strength, a rather low synchronization index would be observed. The potential of the phase difference has no clearly pronounced minimum (Fig. 3(a)).

In the case of the transfer function model under investigation, phase jumps in one direction seem to be preferred as illustrated in Fig. 1(d). This effect can be motivated by the following argument. Imagine that there is no low-pass filter. Then the input Roessler system shows phase trajectories close to the origin of the phase space, leading to phase slips. The output signal, a time-shifted version of the input signal, exhibits phase slips a few steps time-lagged to the input signal. Due to the fact that the additional noise influence in the output signal usually is expected to lead to more phase jumps in u than in x, one preferred direction of phase jumps is observed.

In order to investigate the direction of phase jumps and to test the validity of the hypothesis above, the number of phase jumps in negative direction pj_- and in positive direction pj_+ are counted and a normalized index

$$\Pi = \frac{|pj_+ - pj_-|}{pj_+ + pj_-} \tag{16}$$

is introduced. This index is close to one, if phase jumps in one direction are preferred and zero, if phase jumps in both directions occur with almost the same frequency. If $pj_+ = pj_- = 0$, Π is set to zero. This index is illustrated in Fig. 4(a) for the coupled stochastic Roessler system and Fig. 5(a) for the transfer function model under investigation. In Fig. 4(b) and Fig. 5(b), respectively, corresponding values of the phase synchronization index R (cf. Eq. (10)) are given.

For the coupled stochastic Roessler system the frequency mismatch $\Delta\omega$ between the two oscillators and the bidirectional coupling strength $\varepsilon_{1,2}$ are controlled. High synchronization values for certain parameter ranges in Fig. 4(b) strongly indicates phase synchronization between the two oscillators. The corresponding area belongs to a small frequency mismatch combined with a rather strong coupling. For these parameter values, the number of phase jumps in positive direction is approximately of the same order of magnitude as the number of phase jumps in negative direction and therefore the index Π is close to zero, which confirms the observation in Fig. 1(c). For some combinations of the parameters $\varepsilon_{1,2}$ and $\Delta\omega$, phase jumps in both directions occur but with a higher frequency in one direction. This is indicated by a value of the index $\Pi > 0$. For the area corresponding to a high frequency mismatch and a low coupling strength, phase jumps with one preferred direction occur. This area of parameters, however, corresponds to rather low values of the phase synchronization index.

In the case of the transfer function system the two parameters a_1 and a_2 are varied. Values of the synchronization index R indicate a relationship between the two processes for $a_2 > 0.2$ and almost independent of a_1 (Fig. 5(b)). The parameter a_2 represents the direct influence of the input signal to the output signal. In this area, phase jumps are preferred in one direction. Therefore, the index Π is close to one. But in a small range of parameters a_1 and a_2, the index Π is close to zero while the synchronization index R indicates an interaction between the processes.

5.2. Coherence applied to phase fluctuations

Phase synchronization is a frequency related phenomenon. Therefore, a relation between the phases is expected exclusively at the oscillation frequency for phase synchronizing systems. In contrast, a phase relations between a time-shifted and filtered output signal and its corresponding input signal is expected to be present over a certain range of frequencies. Based on this assumption, coherence analysis is applied to the amplitudes A_1 and A_2 and to the phase fluctuations

$$\Phi^{(1,2)}(t) - \Omega_{1,2}t \tag{17}$$

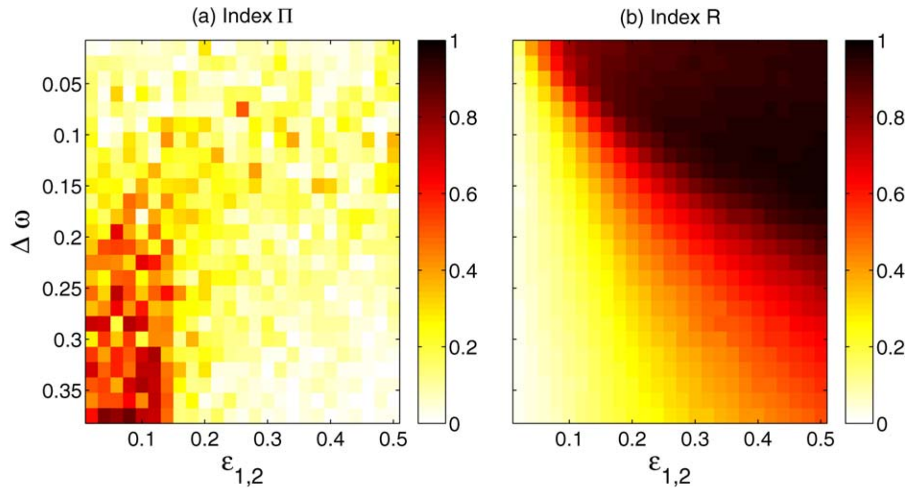

Fig. 4. (a) Index Π (cf. Eq. (16)) for a coupled stochastic Roessler system depending on the bidirectional coupling $\varepsilon_{1,2}$ and frequency mismatch $\Delta\omega$. (b) Values of the corresponding synchronization index R (cf. Eq. (10)). The upper right corner shows high values of R, strongly indicating a preferred value of the phase difference. In the area with high synchronization index R, the index Π quantifying a preferred direction of phase jumps is close to 0. The high values of Π in the lower left area can be neglected since no phase synchronization is detected by the index R.

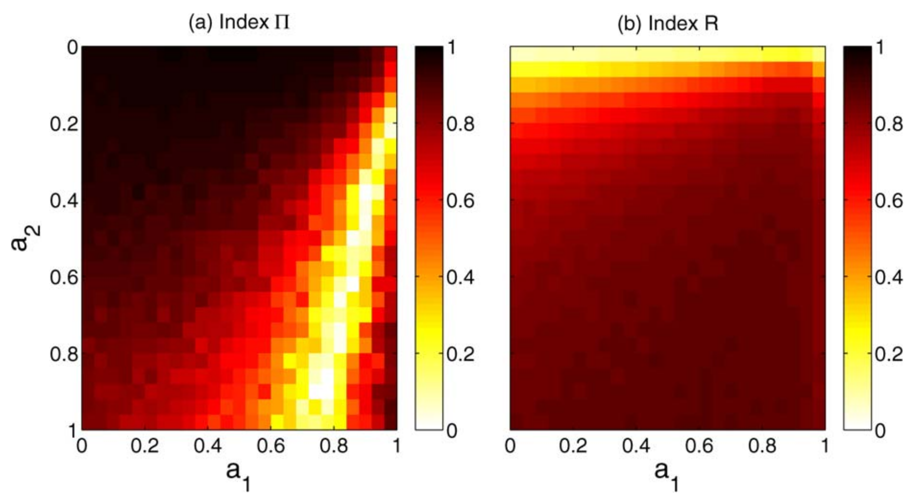

Fig. 5. (a) Index Π (cf. Eq. (16)) for the transfer function system depending on parameters a_1 and a_2. (b) Values of the corresponding synchronization index R (cf. Eq. (10)). If a_2 exceeds a value of 0.2, independent of a_1, high values for the synchronization index are estimated leading erroneously to conclusion of phase synchronization. The index Π indicating a preferred direction of phase jumps is close to one in the area of high values of R. Only in a small area for a_1 between 0.6 and 0.8, the index Π is close to zero.

estimated from the analytic signal representation (cf. Eq. (6)), in order to distinguish between both types of dynamics.

In the following, coherence between the x-components, amplitudes and phase fluctuations for various coupling strengths of the coupled stochastic Roessler system are considered. For a low coupling strength $\varepsilon_{1,2} = 0.008$, no significant coherence can be detected in any case (Fig. 6(a)). This coupling strength corresponds to non-synchronized oscillators in the deterministic case. Increasing the coupling to $\varepsilon_{1,2} = 0.024$ (Fig. 6(b)), which refers to the onset of phase synchronization in the deterministic case, leads to a significant coherence of the x-components at their oscillation frequency $\omega_{1,2}/(2\pi) \approx 0.16$ Hz. Additionally, the amplitudes of the analytic signal A_1 and A_2 (cf. Eq. (6)) are also coherent at this frequency, while their phase fluctuations are still uncorrelated. A further increase of the coupling strength to $\varepsilon_{1,2} = 0.084$ increases the coherence between A_1 and A_2 and between x_1 and x_2 in the low frequency range (Fig. 6(c)). For this coupling strength, there is also a significant coherence between the phase fluctuations at the oscillation frequency. For $\varepsilon_{1,2} = 0.084$, phase synchronization between both oscillators is given in the deterministic case.

In the case of the transfer function system with $a_1 = 0.3$ and $a_2 = 0.7$, a significant coherence over a broad frequency range is detected for all three coherence spectra (Fig. 7). Here, coherence between the phase fluctuations is also significant in the low frequency range. This is different for the coupled stochastic Roessler system, where the coherence spectra yield only significant coherence values at the oscillation frequency but not in the low frequency range. Since there exists an almost constant phase difference between the x-components of the coupled stochastic Roessler system, a significant coherence between the phase fluctuations at the oscillation frequency is

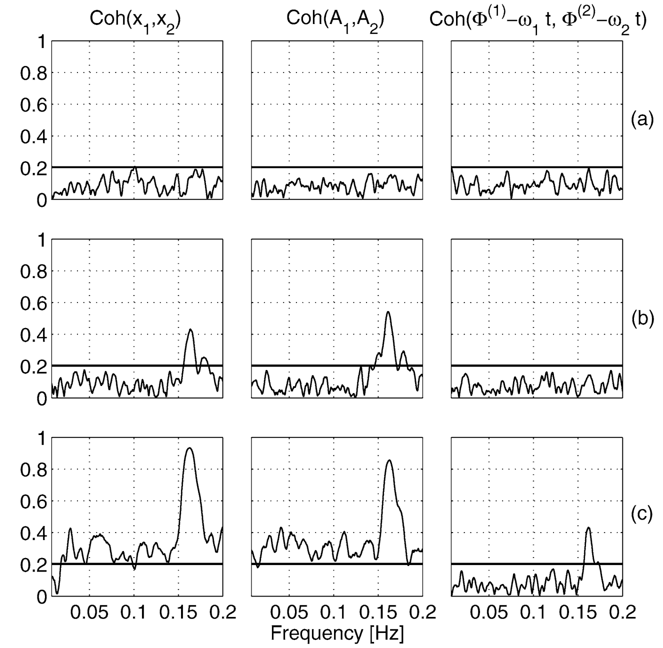

Fig. 6. Coherences estimated between the components x_1 and x_2, the amplitudes of the analytic signal A_1 and A_2 as well as coherence between the phase fluctuations for three coupling strengths $\varepsilon_{1,2}$ of a bidirectionally coupled stochastic Roessler system. The 1%-significance level is given by the solid horizontal lines. (a) For a low coupling strength of $\varepsilon_{1,2} = 0.008$, no significant coherences are detected in neither of these coherence spectra. (b) For $\varepsilon_{1,2} = 0.024$, significant coherences are observed for the x-components and the amplitudes at the oscillation frequency $\omega/2\pi \approx 0.16$ Hz, while the phase fluctuations are still not coherent. (c) Coherences for a coupling strength of $\varepsilon_{1,2} = 0.084$. All three coherence spectra show a significant peak at the oscillation frequency. The coherence between the phase fluctuations is exclusively significant at the oscillation frequency. Additionally, coherence between the components as well as coherence between the amplitudes is significant in the low frequency range.

expected. But at the remaining frequencies, no significant coherences are present as phase synchronization is a frequency related phenomenon. In contrast, highly significant coherences between phase fluctuations are expected for the transfer function system, as the output signal is only the time-shifted and filtered input signal. A time-shift and filter lead to only slight changes in the relationship between the phase signals. Coherence spectra between phase fluctuations yield thus a feature to distinguish between coupled self-sustained oscillatory and transfer function systems.

These results are substantiated for various values of $\Delta\omega$ and $\varepsilon_{1,2}$ or a_1 and a_2 in Fig. 8(a) and (b), respectively. Significant coherence values in a frequency range between 0.02 and 0.12 are averaged. In contrast to the coupled stochastic Roessler system, the average values for the phase fluctuations are highly coherent in case of the transfer function system for $a_1 < 0.8$ and $a_2 > 0.4$.

5.3. *Procedure with high specificity*

We have introduced two different approaches to increase specificity in the previous sections. The corresponding results are summarized in Fig. 8. The upper row shows average coherence values (a), values of the phase synchronization index R (c) and values of the index Π (e) for the coupled stochastic Roessler system. In the lower row, the corresponding values are shown for the transfer function model under investigation (Fig. 8(b), (d), (f)).

To exclude false positive conclusions about the underlying dynamics for phase synchronization and coherence analysis, our investigations suggest the following rule for a sufficiently high synchronization index R: If phase jumps in one direction are preferred *and* phase fluctuations are significantly coherent over a broad range of frequencies, there is strong evidence for a transfer function system. In contrast, if phase jumps have no favorite direction *and* coherence between the phase fluctuations is not significant or only at the oscillation frequency, there is strong evidence for a coupled self-sustained stochastic oscillatory system.

Conclusions about the underlying dynamics have to be taken with care, if results do not clearly fit into one of these two cases. For example, for $a_1 \approx 0.7$ and $a_2 \approx 0.9$ in the transfer function system considered, there is no favorite direction of phase jumps quantified by an index $\Pi \approx 0$ arguing for phase synchronization (Fig. 8(f)). But as phase fluctuations are highly significantly coherent (Fig. 8(b)), a false conclusion to a coupled self-sustained oscillatory system is prevented.

It should be emphasized that the rules derived above do not apply in general. For instance, in the case of almost complete synchronization there are no phase jumps at all and the coherence will be broad-band. Following the above mentioned rules no conclusion is possible, especially no erroneous conclusion is drawn. For other cases, simulation studies as presented in this Letter have to be tailored to the given problem. This is not a weakness of this approach but reflects the challenge of solving an *inverse problem*.

6. Conclusion

Based on results of analysis techniques applied to empirical time series, conclusions about the nature of the underlying system are drawn in many applications. Frequently, the type of underlying dynamics is of particular interest. If analysis techniques are sensitive but not specific in detecting the type of underlying dynamics, conclusions about the dynamic are difficult and sometimes even impossible. We illustrated missing specificity of two widely used analysis techniques by investigations of two representatives of coupled self-sustained oscillators and transfer function systems. Furthermore, conditional on these representatives, we presented extensions to improve specificity based on a detailed analysis of the established methods, coherence and phase synchronization analysis. As a first extension, we investigated the direction of phase jumps. As a second approach, coherence analysis was extended to phase fluctuations estimated from the analytic signal representation.

Given a large phase synchronization index R, examination of the direction of phase jumps and coherence between the phase fluctuations leads to the following rule to increase specificity for the investigated model systems: If phase jumps in one direction are preferred and phase fluctuations are significantly coherent

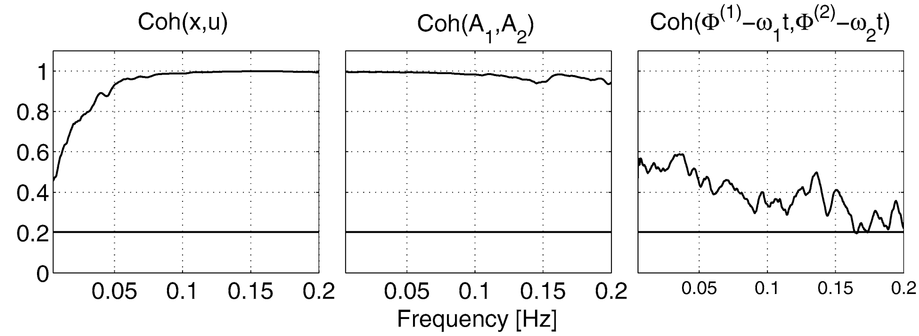

Fig. 7. Coherences between the components x and u, between the amplitudes A_1 and A_2 as well as coherence between the phase fluctuations for the transfer function system. The 1%-significance level is given by the solid horizontal lines. All three coherence spectra are highly significant over a broad frequency range. Especially a significant coherence between the phase fluctuations is detected in the entire low frequency band. This is in contrast to the coupled stochastic Roessler system.

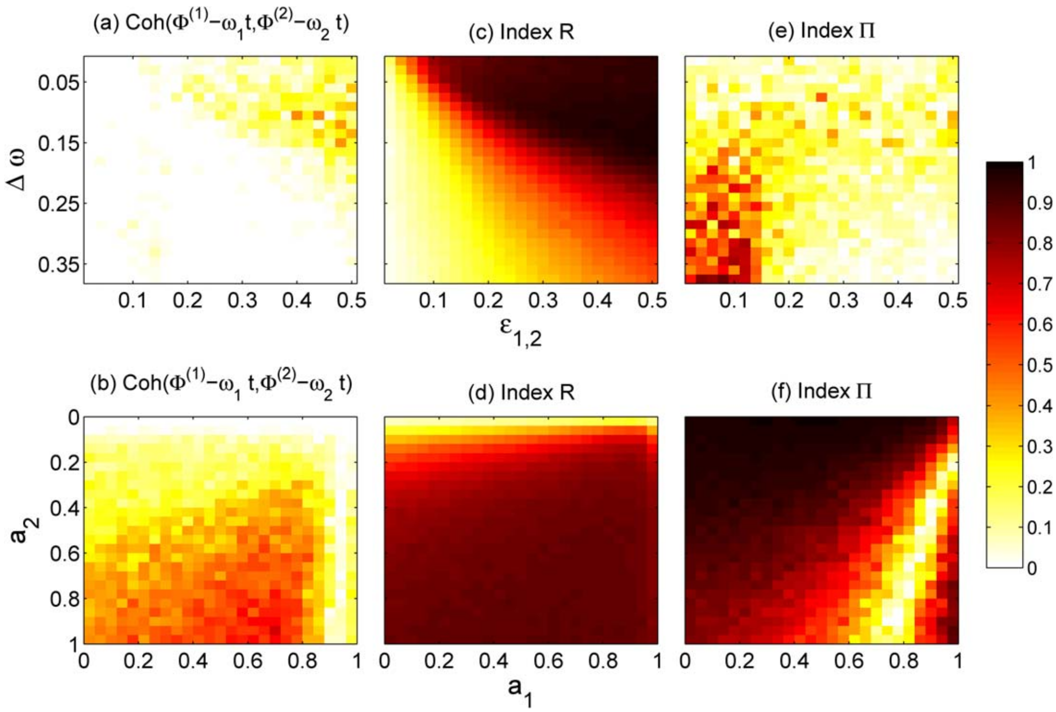

Fig. 8. Averaged coherence values between the phase fluctuations (a), values of the phase synchronization index R (c), and values of the index Π quantifying direction of phase jumps (e) for the coupled stochastic Roessler system. Averaged coherence values between phase fluctuations (b), values of the phase synchronization index R (d), and values of the index Π (f) for the transfer function system. For a high value of the synchronization index R, combination of coherence applied to the phase fluctuations and the direction of phase jumps allows for specific conclusions about the underlying dynamics.

over a broad range of frequencies, there is strong evidence for a transfer function system. In contrast, if phase jumps have no favorite direction and coherence between the phase fluctuations is not significant or only at the oscillation frequency, there is strong evidence for a coupled self-sustained, stochastic oscillatory system. For results which do not clearly fit into one of these two cases, conclusions to the underlying dynamics have to be taken with care for the model systems under investigation.

The restriction to two representatives prevents a general conclusion whether analysis techniques with high specificity could be developed for a larger number of model systems. However, since the proposed adaptations of analysis techniques to increase specificity are based on rather general heuristic theoretical motivations, they are expected to work for a wider class of processes. Further work will be devoted to the investigation of systems such as other transfer function systems and excitable systems [37–39].

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References

[1] D. Brillinger, Time Series: Data Analysis and Theory, Holden–Day, San Francisco, 1981.

[2] H. Kantz, T. Schreiber, Nonlinear Time Series Analysis, Cambridge Univ. Press, Cambridge, 1997.
[3] R. Dahlhaus, Metrika 51 (2000) 157.
[4] L. Baccala, K. Sameshima, Biol. Cybern. 84 (2001) 463.
[5] M. Wiesenfeldt, U. Parlitz, W. Lauterborn, Int. J. Bifur. Chaos 11 (2001) 2217.
[6] S. Schiff, P. So, T. Chang, R. Burke, T. Sauer, Phys. Rev. E 54 (1996) 6708.
[7] S. Boccaletti, J. Kurths, G. Osipov, D. Valladares, C. Zhou, Phys. Rep. 366 (2002) 1.
[8] P. Tass, M.G. Rosenblum, J. Weule, J. Kurths, A. Pikovsky, J. Volkmann, A. Schnitzler, H.J. Freund, Phys. Rev. Lett. 81 (1998) 3291.
[9] M. Paluš, Phys. Lett. A 213 (1996) 138.
[10] M. Paluš, Phys. Lett. A 235 (1997) 341.
[11] M. Paluš, A. Stefanovska, Phys. Rev. E 67 (2003) 055201(R).
[12] M.G. Rosenblum, L. Cimponeriu, A. Bezerianos, A. Patzak, R. Mrowka, Phys. Rev. E 65 (2002) 041909.
[13] B. Hellwig, S. Häußler, B. Schelter, M. Lauk, B. Guschlbauer, J. Timmer, C.H. Lücking, Lancet 357 (2001) 519.
[14] P. Grosse, M. Cassidy, P. Brown, Clin. Neurophysiol. 113 (2002) 1523.
[15] F. Varela, J. Lachaux, E. Rodriguez, J. Martinerie, Nat. Rev. Neurosci. 2 (2001) 229.
[16] P.A. Tass, T. Fieseler, J. Dammers, K. Dolan, P. Morosan, M. Majtanik, F. Boers, A. Muren, K. Zilles, G.R. Fink, Phys. Rev. Lett. 90 (2003) 088101.
[17] M. Breakspear, Hum. Brain Mapp. 15 (2002) 175.
[18] B. Schack, S. Weiss, P. Rappelsberger, Hum. Brain Mapp. 19 (2003) 18.
[19] A. Pikovsky, M. Rosenblum, J. Kurths, Synchronization—A Universal Concept in Nonlinear Sciences, Cambridge Univ. Press, Cambridge, 2001.
[20] M.G. Rosenblum, A.S. Pikovsky, J. Kurths, Phys. Rev. Lett. 76 (1996) 1804.
[21] M.G. Rosenblum, A.S. Pikovsky, J. Kurths, Phys. Rev. Lett. 78 (1997) 4193.
[22] H. Akaike, Some Problems in the Application of the Cross-Spectral Method, in: B. Harris (Ed.), Spectral Analysis of Time Series, Wiley, New York, 1967.
[23] J. Timmer, M. Lauk, W. Pfleger, G. Deuschl, Biol. Cybern. 78 (1998) 359.
[24] O. Rössler, Phys. Lett. A 57 (1976) 397.
[25] J.-P. Lachaux, E. Rodriguez, M. Le van Quyen, A. Lutz, J. Martinerie, F. Varela, Int. J. Bifur. Chaos 10 (2000) 2429.
[26] A.G. Rossberg, K. Bartholome, J. Timmer, Phys. Rev. E 69 (2004) 016216.
[27] D. Gabor, J. IEE London 93 (1946) 429.
[28] A. Oppenheim, R. Schafer, Digital Signal Processing, Prentice–Hall, Englewood Cliffs, NJ, 1975.
[29] B. Boashash, Proc. IEEE 80 (1992) 520.
[30] F. Mormann, K. Lehnertz, P. David, C. Elger, Physica D 144 (2000) 358.
[31] M. Rosenblum, A. Pikovsky, J. Kurths, C. Schäfer, P.A. Tass, Phase synchronization: from theory to data analysis, in: F. Moss, S. Gielen (Eds.), Neuro-Informatics, in: Handbook of Biological Physics, vol. 4, Elsevier, Amsterdam, 2001, pp. 279–321.
[32] M. Priestley, Spectral Analysis and Time Series, Academic Press, London, 1989.
[33] P. Brockwell, R. Davis, Time Series: Theory and Methods, Springer, New York, 1998.
[34] J. Timmer, M. Lauk, W. Pfleger, G. Deuschl, Biol. Cybern. 78 (1998) 349.
[35] D. Halliday, J. Rosenberg, A. Amjad, P. Breeze, B. Conway, S. Farmer, Prog. Biophys. Molec. Biol. 64 (1995) 237.
[36] J. Timmer, M. Lauk, S. Häußler, V. Radt, B. Köster, B. Hellwig, B. Guschlbauer, C. Lücking, M. Eichler, G. Deuschl, Int. J. Bifur. Chaos 10 (2000) 2595.
[37] A. Hodgkin, A. Huxley, J. Physiol. 117 (1952) 500.
[38] R. Fitzhugh, Biophys. J. 1 (1961) 445.
[39] J. Nagumo, S. Arimoto, S. Yoshizawa, Proc. Inst. Radio Eng. 50 (1962) 2061.